

Conjugated gammadion chiral metamaterial with uniaxial optical activity and negative refractive index

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We demonstrate numerically and experimentally a conjugated gammadion chiral metamaterial that uniaxially exhibits huge optical activity and circular dichroism, and gives a negative refractive index. This chiral design provides smaller unit cell size and larger chirality compared with other published planar designs. Experiments are performed at GHz frequencies (around 6 GHz) and are in good agreement with the numerical simulations.

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I. INTRODUCTION

Materials having a magnetic or electric moment that can be excited by the parallel external electric or magnetic field of the incident electromagnetic (EM) wave exhibiting optical activity¹ are called chiral materials. They are characterized by the quantity of *chirality*, $\kappa = (n_R - n_L)/2$, where n_R/n_L is the refractive index of the right- or left-handed circular polarized wave (RCP/LCP). Natural chiral materials have very weak chirality (e.g., for quartz, $\kappa \simeq 5 \times 10^{-5}$ at $\lambda = 400$ nm). Five orders of magnitude stronger chirality can be realized by chiral metamaterials made with subwavelength resonators. Chiral metamaterials recently attracted much interest because of strong chirality,² negative refractive index,^{3,4} and the prospect of a repulsive Casimir force.⁵ Many chiral metamaterial designs have been proposed and demonstrated to obtain large optical activity, circular dichroism,^{6,7} and negative refractive index.⁸⁻¹²

In this article, we study both numerically and experimentally the conjugated gammadion chiral metamaterial that uniaxially exhibits huge optical activity, circular dichroism, and negative refractive index. This new design more clearly lacks any mirror symmetry plane compared to previous work.¹³ In Ref. 13, the structure itself has a mirror symmetry plane. The lack of mirror symmetry only originates from the presence of the substrate and different trace widths in the bilayer structure due to fabrication constraints. In our design, the trace widths are the same and the lack of mirror symmetry is an inherent property of the particular design. Our design also possesses smaller unit cell size and larger chirality compared with other published planar designs, such as twisted rosettes,⁸ twisted crosswires,^{6,10} and four U split ring resonators (SRRs).^{7,14,15}

II. EXPERIMENT AND SIMULATION

The layout of the conjugated gammadion chiral metamaterial is shown in Fig. 1. A 30×30 array of the conjugated gammadion resonator pairs is patterned on each side of the flame-resistant grade 4 (FR-4) board. The relative dielectric constant of the FR-4 board is $\epsilon = 4.2$ with a dielectric loss tangent of 0.02. The two layers of the gammadion

resonators are conjugatedly arranged in order to break the mirror symmetry along the direction perpendicular to the metamaterial plane. The metamaterial possesses C_4 symmetry on the z axis and, therefore, exhibits uniaxial chirality for the normal incident EM wave. The dimensions of the unit cell and the photograph of the experimental sample are shown in Fig. 1.

The amplitudes and phases of the linearly polarized transmissions T_{\parallel} (polarization of the transmitted wave is parallel to that of the incident wave) and T_{\perp} (polarization of the transmitted wave is perpendicular to that of the incident wave), and reflection coefficient R ($R = R_{\parallel}, R_{\perp} = 0$),¹⁶ are measured using an HP 8364B network analyzer with two Narda standard horn antennas. The circular polarized transmissions of the RCP and LCP, T_R and T_L , and reflections, R_R and R_L , can be obtained from the linearly polarized transmission coefficients by

$$T_R = T_{\parallel} - iT_{\perp}, \quad T_L = T_{\parallel} + iT_{\perp}, \quad R_R = R_L = R. \quad (1)$$

The circular polarized waves are the eigenwave functions of the chiral metamaterials. The cross transmissions, from LCP to RCP and vice versa, are zero. The numerical simulations were performed by using the frequency domain solver of the CST Microwave Studio (Computer Simulation Technology GmbH, Darmstadt, Germany), which implements a finite element method. In the simulations, the unit cell boundary condition was applied and the circular polarized eigenwaves were used directly.

III. RESULTS

Figures 2(a) and 2(b) show the simulation (left) and experimental (right) results of the transmissions and reflections. The calculated results nicely agree with our experimental results. There are two resonances in the transmission spectrum. One corresponds to a peak around $f = 5.6$ GHz and the other corresponds to a dip around $f = 7.8$ GHz. The first resonance is much sharper than the second one. The feature of the transmission peak for both the RCP and the LCP at the first resonance is observed in chiral metamaterials. From the following retrieval results, we conclude that this peak is induced

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14. ABSTRACT Materials having a magnetic or electric moment that can be excited by the parallel external electric or magnetic field of the incident electromagnetic (EM) wave exhibiting optical activity¹ are called chiral materials. They are characterized by the quantity of chirality, $\chi = (n_R - n_L)/2$, where n_R/n_L is the refractive index of the right- or left-handed circular polarized wave (RCP/LCP). Natural chiral materials have very weak chirality (e.g., for quartz, $\chi \approx 5 \times 10^{-5}$ at $\lambda = 400$ nm). Five orders of magnitude stronger chirality can be realized by chiral metamaterials made with subwavelength resonators. Chiral metamaterials recently attracted much interest because of strong chirality,² negative refractive index,^{3,4} and the prospect of a repulsive Casimir force.⁵ Many chiral metamaterial designs have been proposed and demonstrated to obtain large optical activity, circular dichroism,^{6,7} and negative refractive index.					
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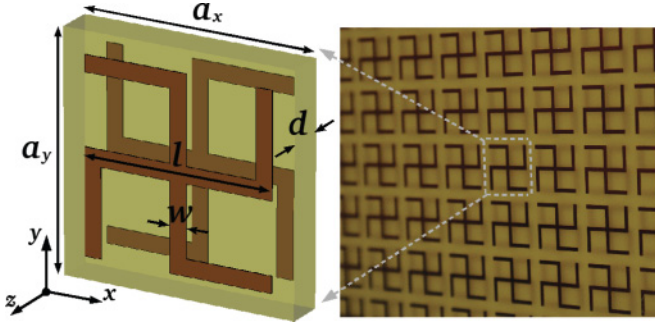


FIG. 1. (Color online) Scheme of the conjugated gammadion chiral metamaterial. Two gammadion copper resonators are conjugatedly aligned on each side of the FR-4 board. The geometry parameters are given as $a_x = a_y = 10$ mm, $l = 8.1$ mm, $w = 0.7$ mm, and $d = 1.6$ mm, and the thickness of copper is $t = 0.036$ mm.

by the strong magnetic response, which decreases the degree of impedance mismatch. At the resonances, the transmission spectra for the RCP and LCP waves are significantly different. At the first resonance, the transmission of the LCP is larger than that of the RCP; at the second resonance, it reverses. This difference between the amplitudes of two transmissions is characterized by ellipticity, $\eta = \frac{1}{2} \tan^{-1} \left(\frac{|T_L|^2 - |T_R|^2}{|T_L|^2 + |T_R|^2} \right)$, as shown in Figs. 2(e) and 2(f). Far from the resonances, the phases of the RCP and LCP converge. In the vicinity of the resonances, they are obviously different, which will induce the rotation of the polarization plane of a linearly polarized light as it passes through the chiral metamaterial. The difference between the phases is characterized by the rotary power,

$\theta = \frac{1}{2} [\arg(T_L) - \arg(T_R)]$, as shown in Figs. 2(c) and 2(d). The most interesting region is around $\eta = 0$, which will give the “pure” optical activity effect (i.e., for the linearly polarized incident wave, the transmission wave will still be linearly polarized, just with the polarization plane rotated by an angle of θ). This design possesses very strong optical activity. At $\eta = 0$ (i.e., $f = 6.55$ GHz), the rotation angle is about 30° for 1.6-mm-thick metamaterial (i.e., the rotation angle is as large as $856^\circ/\lambda$).

After obtaining the data of the transmissions and reflections, we can apply a retrieval procedure to obtain the effective constitutive parameters.^{17,18} The chirality κ can be obtained directly from the transmissions as

$$\text{Re}(\kappa) = \frac{\arg(T_L) - \arg(T_R) + 2m\pi}{2k_0d}, \quad (2a)$$

$$\text{Im}(\kappa) = \frac{\ln |T_L| - \ln |T_R|}{2k_0d}, \quad (2b)$$

where k_0 is the wave vector in the vacuum, d is the thickness of the sample (here, the thickness of the retrieved sample is chosen as 1.6 mm, that is, the thickness of the FR-4 board), and the integer m is determined by the condition of $-\pi < \arg(T_L) - \arg(T_R) + 2m\pi < \pi$ for one unit cell. In previous studies,^{19,20} the retrieved parameters depend on the thickness of the sample¹⁹ and once we have the multilayer sample, the retrieved parameters, for the weakly coupled system, are very close to the one unit cell and converge very quickly.²⁰ For the strongly coupled system, the retrieved parameters are completely different from the one unit cell system.^{20,21}

Comparing with Figs. 2(c)–2(f), we note that the real and imaginary parts of κ relate to the azimuth rotation angle θ and the circular dichroism η , respectively. The average refractive index $n = (n_L + n_R)/2$ and the impedance z can be obtained by using the traditional retrieval procedure¹⁸ after taking the geometric average transmission $T = \sqrt{T_L T_R}$. The branch of the square root is chosen coordinating with the real part of κ . Then, the other parameters can be calculated by $n_{\pm} = n \pm \kappa$, $\epsilon = n/z$, and $\mu = nz$. Figure 3 shows the retrieval results. The chirality is very large, $\kappa = 2.35$ at $\eta = 0$, which corresponds to frequency $f = 6.55$ GHz ($\kappa = 2.09$ for the experimental result). This strong chirality can easily push the refractive index of the RCP/LCP eigenwave, $n_{\pm} = n \pm \kappa$, to become negative at the resonances, as shown in Figs. 3(c) and 3(d). Due to the loss of the FR-4 board, the figure of merit (FOM), $-\text{Re}(n)/\text{Im}(n)$, is very low compared with the traditional negative index metamaterials.²² If we use a lower loss substrate, the FOM can be larger than 10 according to our numerical simulation. In Figs. 3(e) and 3(f), we show the retrieved real parts of the permittivity and permeability of the chiral metamaterials.

IV. DISCUSSION

To illustrate the origin of the chiral response of our metamaterial, in Fig. 4, we discuss a procedure of transmutation from the simple Ω -particle chiral element to the conjugated gammadion chiral metamaterial. The Ω -particle chiral element is one of the important chiral structures studied analytically

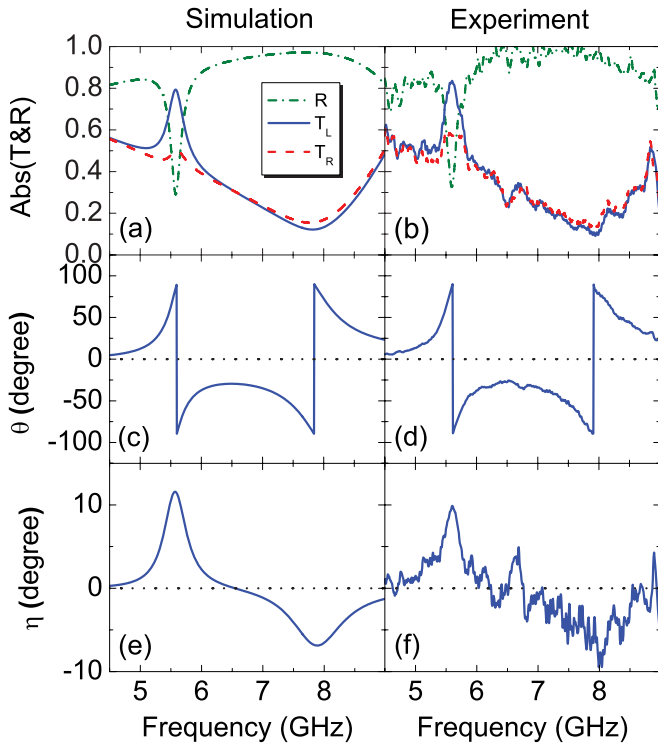


FIG. 2. (Color online) Simulation (left) and experimental (right) results of the transmissions and reflections, and the rotation angle and circular dichroism.

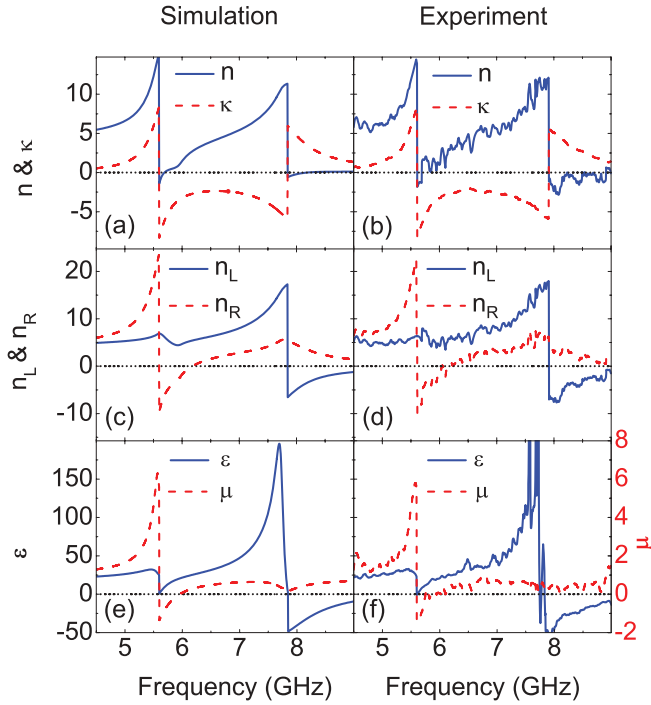


FIG. 3. (Color online) Retrieval results from the numerical (left) and experimental (right) results of the transmissions and reflections.

elsewhere.¹ Here, we place two Ω -particle chiral elements together to form a conjugated Ω -particle chiral element pair. The linearly polarized external electric field can drive the electric dipole via the vertical arms. This electric response (ER) generates circular electric current on the loop that gives a magnetic moment in the same direction as the electric dipole. By rotating the conjugated Ω -particle chiral element pair by 90° , the external magnetic field will drive the magnetic moment via the loop. This magnetic response (MR) then

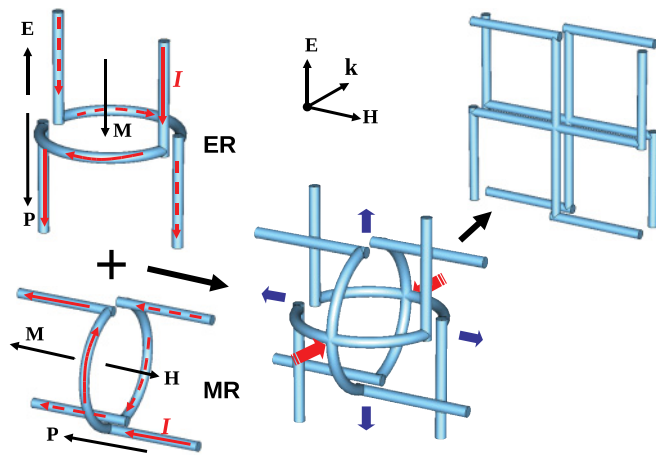


FIG. 4. (Color online) Procedure of the transmutation from the simple Ω -particle chiral element to the conjugated gammadion chiral metamaterial. The procedure shows how the magnetic or electric moment is induced by the electric or magnetic field (of the incident EM wave) in the parallel direction. ER and MR are the electric and magnetic response modes, respectively. The arrow indicates the current direction.

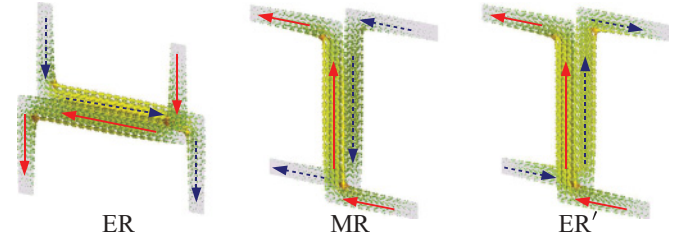


FIG. 5. (Color online) Three basic electromagnetic responses in conjugated gammadion chiral metamaterials. The arrow indicates the current direction. ER and MR are comparable with those in Fig. 4. ER' is an emerged electric response that is driven by the central arms. ER' cannot give chirality response. The incident wave is the same as in Fig. 4.

drives the current on the vertical arms, giving an electric dipole in the same direction as the magnetic moment. This clearly shows how the magnetic or electric moment is induced by the electric or magnetic field (of the incident EM wave) in the parallel direction. According to the aforementioned definition, this results in the chirality. If we add the two Ω -particle pairs together, it will be a bi-isotropic uniaxial chiral metamaterial. After stretching the semicircle loop to a straight line, it becomes our current planar design.

To further illustrate the origin of chirality in our conjugated gammadion chiral metamaterials, Fig. 5 shows three basic electromagnetic responses (ER, MR, and ER') in conjugated gammadion chiral metamaterials. ER and MR are comparable with those in Fig. 4. ER' is an additional, nonresonant electric response that is driven by the central parallel arms. ER' cannot give chirality response because there is no magnetic moment generated. In our conjugated gammadion chiral metamaterials, each resonance is a superposition of these three electromagnetic response modes. For example, as shown in Fig. 6, the resonance at $f = 5.6$ GHz can be considered as the superposition of ER, MR, and ER'. The existence of the

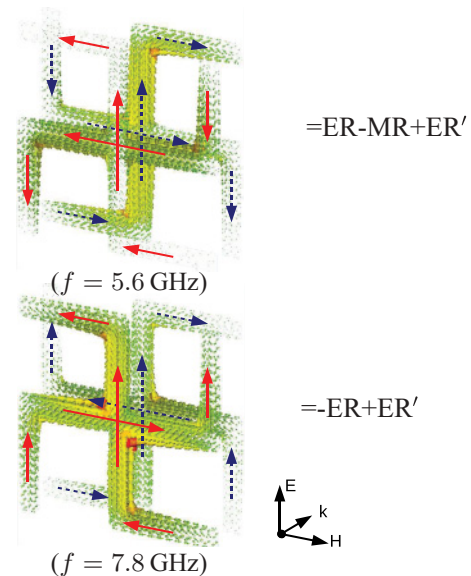


FIG. 6. (Color online) Illustration of superposition at $f = 5.6$ GHz (upper) and $f = 7.8$ GHz (lower).

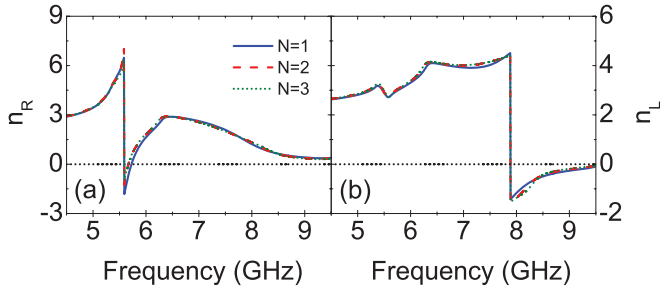


FIG. 7. (Color online) Retrieved refractive indices for multilayer conjugated gammadion chiral metamaterial.

MR is also reflected by the retrieval results in Figs. 3(e) and 3(f), where the effective permeability has a strong resonance at $f = 5.6$ GHz. In addition, the resonance at $f = 7.8$ GHz can be considered as the superposition of ER and ER'; then the retrieval results only show the electric response.

Some may raise the question of whether our design is just a thin metasurface that exhibits rotation of polarization phase or can represent the properties of the bulk chiral metamaterial as we expected. In order to clarify this point, in Fig. 7 we plot the retrieved refractive indices for multilayer metamaterial (a pair of double-layered conjugated gammadions is called one layer). Here, we study the weakly coupling case:²⁰ The separation D between each layer of conjugated gammadion pairs is three times the thickness of the FR-4 board (i.e., $D = 3d$) and the total thickness of each unit cell is $D + d = 6.4$ mm. Figure 7 shows that the retrieved refractive indices for the multilayer metamaterial are rapidly converged. Three layers ($N = 3$) almost overlap with two layers ($N = 2$). Therefore, the single layer we studied previously can represent the properties of

bulk chiral metamaterials for these weakly coupling cases. The differences between the refractive indices in Figs. 3 and 7 for the single unit cell are due to the different thicknesses of the unit cell. In Fig. 3, we take the unit cell thickness as 1.6 mm, while it is 6.4 mm in Fig. 7. Comparing Fig. 7 and Figs. 3(e) and 3(f), the magnitude of the retrieved effective parameters of n_R and n_L decreases, as the size of the unit cell increases.¹⁹ Note that, for the strong coupling cases (i.e., small D), the strong coupling between each layer will induce differences between one layer and multilayer structure²¹ and slow the convergence of the retrieved parameters.²⁰

V. CONCLUSION

In conclusion, we have designed and studied a conjugated gammadion chiral metamaterial at around 6 GHz. This chiral metamaterial exhibits huge uniaxial optical activity. The rotation angle is as large as 30° for a 1.6-mm-thick metamaterial, corresponding to $\kappa = 2.35$ with $\eta = 0$. The chiral metamaterial also exhibits uniaxial negative refractive index, due to its strong chirality. The origin of the chirality is intuitively and physically explained, based on the Ω -particle chiral element model.

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¹I. V. Lindell, A. H. Sihvola, S. A. Tretyakov, and A. J. Vitanen, *Electromagnetic Waves in Chiral and Bi Isotropic Media* (Artech House, Boston, 1994).

²A. V. Rogacheva, V. A. Fedotov, A. S. Schwanecke, and N. I. Zheludev, *Phys. Rev. Lett.* **97**, 177401 (2006).

³J. B. Pendry, *Science* **306**, 1353 (2004).

⁴S. Tretyakov, A. Sihvola, and L. Jylhä, *Photonics Nanostruct. Fundam. Appl.* **3**, 107 (2005).

⁵R. Zhao, J. Zhou, Th. Koschny, E. N. Economou, and C. M. Soukoulis, *Phys. Rev. Lett.* **103**, 103602 (2009).

⁶M. Decker, M. Ruther, C. E. Kriegler, J. Zhou, C. M. Soukoulis, S. Linden, and M. Wegener, *Opt. Lett.* **34**, 2501 (2009).

⁷M. Decker, R. Zhao, C. M. Soukoulis, S. Linden, and M. Wegener, *Opt. Lett.* **35**, 1593 (2010).

⁸E. Plum, J. Zhou, J. Dong, V. A. Fedotov, Th. Koschny, C. M. Soukoulis, and N. I. Zheludev, *Phys. Rev. B* **79**, 035407 (2009).

⁹S. Zhang, Y. S. Park, J. Li, X. Lu, W. Zhang, and X. Zhang, *Phys. Rev. Lett.* **102**, 023901 (2009).

¹⁰J. Zhou, J. Dong, B. Wang, Th. Koschny, M. Kafesaki, and C. M. Soukoulis, *Phys. Rev. B* **79**, 121104(R) (2009).

¹¹B. Wang, J. Zhou, Th. Koschny, and C. M. Soukoulis, *Appl. Phys. Lett.* **94**, 151112 (2009).

¹²B. Wang, J. Zhou, Th. Koschny, M. Kafesaki, and C. M. Soukoulis, *J. Opt. A: Pure Appl. Opt.* **11**, 114003 (2009).

¹³M. Decker, M. W. Klein, M. Wegener, and S. Linden, *Opt. Lett.* **32**, 856 (2007).

¹⁴X. Xiong, W. H. Sun, Y. J. Bao, M. Wang, R. W. Peng, C. Sun, X. Lu, J. Shao, Z. F. Li, and N. B. Ming, *Phys. Rev. B* **81**, 075119 (2010).

¹⁵Z. Li, R. Zhao, Th. Koschny, M. Kafesaki, K. B. Alici, E. Colak, H. Caglayan, E. Ozbay, and C. M. Soukoulis, *Appl. Phys. Lett.* **97**, 081901 (2010).

¹⁶A. Lakhtakia, V. V. Varadan, and V. K. Varadan, *J. Opt. Soc. Am. A* **7**, 1654 (1990).

¹⁷R. Zhao, Th. Koschny, and C. M. Soukoulis, *Opt. Express* **18**, 14553 (2010).

¹⁸D. R. Smith, S. Schultz, P. Markoš, and C. M. Soukoulis, *Phys. Rev. B* **65**, 195104 (2002).

¹⁹J. Zhou, Th. Koschny, M. Kafesaki, and C. M. Soukoulis, *Photon Nanostruct. Fundam. Appl.* **6**, 96 (2008).

²⁰J. Zhou, Th. Koschny, M. Kafesaki, and C. M. Soukoulis, *Phys. Rev. B* **80**, 035109 (2009).

²¹Z. Li, H. Caglayan, E. Colak, J. Zhou, C. M. Soukoulis, and E. Ozbay, *Opt. Express* **18**, 5375 (2010).

²²C. M. Soukoulis, J. Zhou, Th. Koschny, M. Kafesaki, and E. N. Economou, *J. Phys.: Condens. Matter* **20**, 304217 (2008).